9.5a More Tests for Convergence

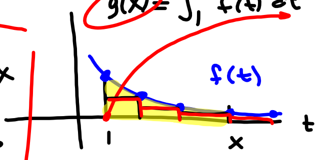
The Integral Test

$\sum_{n=1}^{\infty} f(n)$ & $\int_1^{\infty} f(x)\,dx$

behave the same

both converge or diverge

$g(x) = \int_1^x f(t)\,dt$

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Does the series converge?

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$$

converges by integral test

$$\int_1^{\infty} \frac{1}{x\sqrt{x}}\,dx = \int_1^{\infty} \frac{1}{x^{3/2}}\,dx$$

converges to 2

$$\lim_{b\to\infty} \int_1^b x^{-3/2}\,dx = \lim_{b\to\infty} \left(-2x^{-\frac{1}{2}}\right|_1^b$$

$$= \lim_{b\to\infty} \frac{-2}{\sqrt{b}} - \frac{-2}{\sqrt{1}} = 2$$

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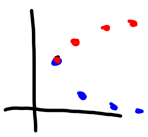
Harmonic Series and P-Series

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ $\quad p > 1$ series converges

$\quad p \le 1$ series diverges

$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} \ldots$

harmonic series diverges



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Limit Comparison Test

$\lim_{n\to\infty} \frac{a_n}{b_n} = c \neq 0$

If $a_n$ & $b_n$ grow at the same rate

then $\sum a_n$ & $\sum^{\infty} b_n$ behave the same

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Do the following series converge or diverge?

$$\sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2} \qquad \sum \frac{2n}{n^2} = \sum \frac{2}{n}$$

diverges

$$= 2\sum_{n=1}^{\infty} \frac{1}{n}$$

harmonic diverges

$$\lim_{n\to\infty} \frac{2n+1}{(n+1)^2} \cdot \frac{n}{2}$$

$$\lim_{n\to\infty} \frac{2n^2+\cdots}{2n^2+\cdots} = 1$$

seq. grow at same rate

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$$\sum_{n=1}^{\infty} \frac{1}{2^n - 1} \qquad \sum_{n=1}^{\infty} \frac{1}{2^n} \quad \text{geo } r = \frac{1}{2}$$

converges by LCT

converges

$$\lim_{n\to\infty} \frac{1}{2^n-1} \cdot \frac{2^n}{1} = 1$$

seq. grow at same rate

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$$\sum_{n=2}^{\infty} \frac{3n+2}{n^3 - 2n}$$

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